

Complex Scaling in Neutrino Mass Matrix

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Abstract

Using the residual symmetry approach, we propose a complex extension of the scaling ansatz on M_ν which allows a nonzero mass for each of the three light neutrinos as well as a nonvanishing θ_{13} . Leptonic Dirac CP violation must be maximal while atmospheric neutrino mixing need not to be exactly maximal. Each of the two Majorana phases, to be probed by the search for $0\nu\beta\beta$ decay, has to be zero or π and a normal neutrino mass hierarchy is allowed.

If $G_i^T M_\nu G_i = M_\nu$ defines a horizontal symmetry for the complex symmetric M_ν and $U^T M_\nu U = M_d$, where M_d has only real positive diagonal nondegenerate elements, then another unitary matrix $V = Ud$ also puts M_ν into a diagonal form, where $d = \text{diag}(d_1, d_2, d_3)$ with $d_{i(i=1,2,3)} = \pm 1$. Moreover, $U^\dagger G_i U = d_i$. Each d_i defines a Z_2 symmetry and the corresponding G_i is also a representation of that Z_2 symmetry. Among eight possible forms of d_i , only two can be shown to be independent, taken as $d_2 = \text{diag}(-1, 1, -1)$, $d_3 = \text{diag}(-1, -1, 1)$. Thus the two independent representations $G_{2,3}$ describe a residual $Z_2 \times Z_2$ flavor symmetry [1, 2] in M_ν . In this way we reinterpret the Simple Real Scaling ansatz [3] in M_ν as a $Z_2 \times Z_2$ symmetry. We further make a complex extension of this invariance and obtain the corresponding M_ν . Interesting phenomenological consequences follow. Here we sketch our method and present the basic results leaving many details to a future lengthier publication. Throughout we follow the PDG convention.

The Simple Real Scaling ansatz [3] attributes the following structure to the neutrino mass matrix

$$M_\nu^{SRS} = \begin{pmatrix} X & -Yk & Y \\ -Yk & Zk^2 & -Zk \\ Y & -Zk & Z \end{pmatrix} \quad (1)$$

with X, Y, Z as complex mass dimensional quantities and k as a real positive dimensionless scaling factor. It has one vanishing mass eigenvalue with the corresponding eigenvector $(0, \frac{e^{i\frac{\beta}{2}}}{\sqrt{1+k^2}}, \frac{ke^{i\frac{\beta}{2}}}{\sqrt{1+k^2}})^T$. The mixing matrix is

$$U^{SRS} = \begin{pmatrix} c_{12} & s_{12}e^{i\frac{\alpha}{2}} & 0 \\ -\frac{ks_{12}}{\sqrt{1+k^2}} & \frac{kc_{12}e^{i\frac{\alpha}{2}}}{\sqrt{1+k^2}} & \frac{e^{i\frac{\beta}{2}}}{\sqrt{1+k^2}} \\ \frac{s_{12}}{\sqrt{1+k^2}} & -\frac{c_{12}e^{i\frac{\alpha}{2}}}{\sqrt{1+k^2}} & \frac{ke^{i\frac{\beta}{2}}}{\sqrt{1+k^2}} \end{pmatrix} \quad (2)$$

with an arbitrary θ_{12} and Majorana phases α, β . Now $G_{2,3}$ can be calculated from $Ud_{2,3}U^\dagger$ to be

$$G_2^k = \begin{pmatrix} -\cos 2\theta_{12} & \frac{k \sin \theta_{12}}{\sqrt{1+k^2}} & -\frac{\sin \theta_{12}}{\sqrt{1+k^2}} \\ \frac{k \sin \theta_{12}}{\sqrt{1+k^2}} & \frac{k^2 \cos 2\theta_{12} - 1}{1+k^2} & \frac{-k(\cos 2\theta_{12} + 1)}{1+k^2} \\ -\frac{\sin \theta_{12}}{\sqrt{1+k^2}} & \frac{-k(\cos 2\theta_{12} + 1)}{1+k^2} & \frac{\cos 2\theta_{12} - k^2}{1+k^2} \end{pmatrix}, G_3^{scaling} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1-k^2}{1+k^2} & \frac{2k}{1+k^2} \\ 0 & \frac{2k}{1+k^2} & \frac{k^2-1}{1+k^2} \end{pmatrix}. \quad (3)$$

The form of U^{SRS} in (2) implies a vanishing s_{13} . Since this has been experimentally excluded at $> 10\sigma$, the SRS ansatz has to be discarded. However, we shall retain G_2^k as well as $G_3^{scaling}$ and propose a complex extension. Our complex extension postulates

$$(G_3^{scaling})^T (M_\nu)^{CES} G_3^{scaling} = (M_\nu^{CES})^*. \quad (4)$$

The corresponding mass matrix M_ν^{CES} can be deduced to be

$$M_\nu^{CES} = \begin{pmatrix} x & -y_1 k + i \frac{y_2}{k} & y_1 + i y_2 \\ -y_1 k + i \frac{y_2}{k} & z_1 - w_1 \frac{k^2-1}{k} - i z_2 & w_1 - i \frac{k^2-1}{2k} z_2 \\ y_1 + i y_2 & w_1 - i \frac{k^2-1}{2k} z_2 & z_1 + i z_2 \end{pmatrix}, \quad (5)$$

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where x, y_1, y_2, z_1, z_2 and w are real mass dimensional quantities. Eq.(4) implies $U^\dagger G_3 U^* = \tilde{d}$ or,

$$G_3 U^* = U \tilde{d}. \quad (6)$$

Once again, $\tilde{d}_{lm} = \pm \delta_{lm}$ if the neutrino masses $m_{1,2,3}$ are all nondegenerate. The LHS of (6) can be written out as

$$\begin{pmatrix} -(U_{e1}^{CES})^* & -(U_{e2}^{CES})^* & -(U_{e3}^{CES})^* \\ \frac{1-k^2}{1+k^2}(U_{\mu 1}^{CES})^* + \frac{2k}{1+k^2}(U_{\tau 1}^{CES})^* & \frac{1-k^2}{1+k^2}(U_{\mu 2}^{CES})^* + \frac{2k}{1+k^2}(U_{\tau 2}^{CES})^* & \frac{1-k^2}{1+k^2}(U_{\mu 3}^{CES})^* + \frac{2k}{1+k^2}(U_{\tau 3}^{CES})^* \\ \frac{2k}{1+k^2}(U_{\mu 1}^{CES})^* - \frac{1-k^2}{1+k^2}(U_{\tau 1}^{CES})^* & \frac{2k}{1+k^2}(U_{\mu 2}^{CES})^* - \frac{1-k^2}{1+k^2}(U_{\tau 2}^{CES})^* & \frac{2k}{1+k^2}(U_{\mu 3}^{CES})^* - \frac{1-k^2}{1+k^2}(U_{\tau 3}^{CES})^* \end{pmatrix}. \quad (7)$$

The reality of $(U_{PMNS})_{e1}$ rules out the possibility $(\tilde{d}_i)_{11} = 1$. Now there are four cases: $\tilde{d}_a \equiv \text{diag. } (-1, 1, 1)$, $\tilde{d}_b \equiv \text{diag. } (-1, 1, -1)$, $\tilde{d}_c \equiv \text{diag. } (-1, -1, 1)$, $\tilde{d}_d \equiv \text{diag. } (-1, -1, -1)$.

These structures of \tilde{d} and the use of (6) lead to the equations given in the following table.

Elements of U^{CES}	Constraint conditions
$\mu 1$	$2kU_{\mu 1}^{CES} = (1 - k^2)U_{\tau 1}^{CES} - (1 + k^2)(U_{\tau 1})^*$
$\tau 1$	$2kU_{\tau 1}^{CES} = -(1 - k^2)U_{\mu 1}^{CES} - (1 + k^2)(U_{\mu 1})^*$
$\mu 2$	$2kU_{\mu 2}^{CES} = (1 - k^2)U_{\tau 2}^{CES} + \eta(1 + k^2)(U_{\tau 2})^*$
$\tau 2$	$2kU_{\tau 2}^{CES} = -(1 - k^2)U_{\mu 2}^{CES} + \eta(1 + k^2)(U_{\mu 2})^*$
$\mu 3$	$2kU_{\mu 3}^{CES} = (1 - k^2)U_{\tau 3}^{CES} + \xi(1 + k^2)(U_{\tau 3})^*$
$\tau 3$	$2kU_{\tau 3}^{CES} = -(1 - k^2)U_{\mu 3}^{CES} + \xi(1 + k^2)(U_{\mu 3})^*$

These equations lead to the result that (1) for case a, $\alpha = \pi, \beta = 0$, (2) for case b, $\alpha = \pi, \beta = \pi$, (3) for case c, $\alpha = 0, \beta = 0$ and (4) for case d, $\alpha = 0, \beta = \pi$. Further, $\cos \delta = 0$ where δ is the Dirac phase in U_{PMNS} . In addition, we have the prediction $\tan \theta_{23} = k^{-1}$ which implies that the atmospheric mixing angle need not be strictly maximal. We have taken the 3σ ranges [4] for the quantities $|\Delta m_{31}^2|, \Delta m_{21}^2, \theta_{12}, \theta_{23}, \theta_{13}$ for our phenomenological analysis. We also take the upper bound 0.23 eV on the sum of the light neutrino masses.

Our conclusions are the following:

- 1) Both types of neutrino mass hierarchy are now allowed.
- 2) For normal hierarchy, the lightest mass m_1 ranges from 10^{-4} eV to 0.07 eV and for inverted hierarchy the lightest mass m_3 ranges from 10^{-4} eV to 0.068 eV.
- 3) For both hierarchies, the quantity $|m_{ee}|$ of relevance to $0\nu\beta\beta$ decay can reach upto the value 0.14 eV which will be probed by GERDA phase II data.

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